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Modeling vertical structure in circular velocity of spiral galaxy NGC 4244

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ABSTRACT
We study the vertical gradient in azimuthal velocity of spiral galaxy NGC 4244 in a thin disk model. With surface density accounting for the rotation curve, we model the gradient properties in the approximation of quasi-circular orbits and find the predictions to be consistent with the gradient properties inferred from measurements. This consistency may suggest that the mass distribution in this galaxy is flattened.

Key words: galaxies: spiral – galaxies: individual: NGC 4244 – galaxies: structure – galaxies: kinematics and dynamics

1 INTRODUCTION

Measurements of galactic kinematics carries the information of how gravitating mass is distributed in galactic interiors. A basic quantity encoding this information is a rotation curve. But for a spiral galaxy appropriately aligned with respect to the observer, it is also possible to ascertain and study the vertical structure of the rotation above the mid-plane which provides a further piece of information indispensable for a more precise determination of the galactic mass distribution.

Measurements enabling a more detailed detection of the velocity field structure have been performed, among others, for spiral galaxies: NGC 891 (Oosterloo et al. 2007; Heald et al. 2006), NGC 4559 (Barbieri et al. 2005), NGC 4302 (Heald et al. 2007), NGC 5775 (Heald et al. 2006), for NGC 2403 (Fraternali et al. 2002) and for the Milky Way galaxy (Levine et al. 2008). In particular, it has been established that the azimuthal component of the rotation gradually diminishes in almost a linear fashion with the altitude above the mid-plane.

Recently, we tested the hypothesis that spiral galaxies, in contrast to what seems to be true for most galaxies, do not necessarily have to be dominated by massive non-baryonic dark matter halos – at least it cannot be excluded that some of these galaxies might be flattened disk-like objects. But, in the absence of dominating spherical components, the motion of matter in the galactic mid-plane vicinity, to within the approximation of axial symmetry, should be satisfactorily well described as if governed entirely by the gravitational field of a thin (or a finite-width) axi-symmetric disk with surface (or column) density accounting for the observed fragment of the rotation curve. From a good mass model it is additionally required that its predictions should be consistent with other observations, in particular, the model should predict correct behavior in the structure of the galactic rotation field. In this context we may use the vertical gradient observable as a test of the disk model of spiral galaxies.

We dealt with the vertical gradient in disk model in (Jałocha et al. 2010) and (Jałocha et al. 2011). To describe the gradient in this model we additionally assumed that the projections of orbits onto the mid-plane were approximately circular, which is the subject of the so called *quasi-circular orbits approximation* assumed also in the present paper. In (Jałocha et al. 2010) we also verified our approach by performing a numerical simulation of motion of test bodies on various orbits in the gravitational potential of a thin disk and we compared the analytical estimates of our approach with the simulated fall-off in azimuthal velocity near the mid-plane. The results of the simulation and of our approach were consistent with each other. In the thin disk model framework we were able to reproduce the observed values of the gradient and its properties, such as a linear decrease of azimuthal velocity with the altitude above the mid-plane. We consider this as a success of the thin disk model.

Furthermore, in (Jałocha et al. 2014) we applied a more general, finite-width disk model to study the vertical gradient in the Milky Way. The width of the disk was constrained so as to obtain the best fit to microlensing measurements being another source of information about the mass distribution, quite independent of the rotation curve. On this occasion we had the opportunity to test limits on the applicability of the thin disk to modeling the vertical gradient of rotation. The conclusion was intuitively clear: when the gradient measurements were performed not too close to the

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mid-plane, outside the main concentration of mass of the finite-width disk (for heights at least 0.5 kpc in the case of the Milky Way galaxy), then the approximation of the infinitesimally thin disk was sufficient and there was no need for using the computationally more demanding finite-width disk.

The two observations above are the basis for our use of the thin disk modeling of the vertical gradient in the present paper. We expect that the approach could be applicable also to other flattened galaxies.

NGC 4244 is another galaxy for which the vertical structure in the rotational velocity can be studied. Zschaechner et al. (2011) published a velocity field above the mid-plane as well as the resulting rotation curve in both galactic halves. The reported results for the detected gradient in their numerical model of NGC 4244 were comparable in both galactic halves: $-9^{+3}_{-2}\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ in the approaching half and $-9^{+2}_{-2}\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ in the receding half, with the gradient decreasing in magnitude to $-5^{+2}_{-2}\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ and $-4^{+2}_{-2}\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ near a radius of 10 kpc in these halves, respectively. The authors pointed out that the lag in HI was present in absence of an HI halo. This is an indication that the mass distribution in NGC 4244 may be flattened, and is what motivates our applying of the thin disk to model this particular galaxy. We want to check whether our simple model applied to NGC 4244 would account for the reported gradient values and their variability with the radial distance.

2 MASS DISTRIBUTION IN THIN DISK

In disk model we are looking for a surface density accounting for the entire observed rotation curve under the assumption of axial and mid-plane-reflection symmetries.

For obvious reasons, rotation curves do not extend far enough and one has to extrapolate the rotation data. But then, the model results unavoidably depend on the way one chooses to extrapolate beyond the last measured point. This feature is not a failure of a mass model as such but is inherent in the gravitation of flattened mass distributions. In general, for a flattened mass distribution (if not representable as a union of contiguous similar homeoids), the rotation value at a given radius is depended also on the masses distributed exterior to that radius, likewise, column mass density at a given point is a point-dependent integral functional of the entire rotation curve. Therefore, the thin disk, as a model of a flattened mass distribution, must be used with due care. A more detailed discussion of these issues were described by Bratek et al. (2008). Sometimes, it is possible to find a global solution by iterations when additional measurements complementary to the rotation data are known beyond the last measurement of the rotation (Jałocha et al. 2008).

2.1 Rotation curve of NGC 4244

As for NGC 4244, we take the rotation curve as the starting point. We make use of the rotation measurements published by Zschaechner et al. (2011). They were carried out separately for both halves of the galaxy, but we have superimposed and averaged out these data because the modeling should be done consistently with the axial symmetry the disk model approximation postulates. This approximation should suffice to model the vertical gradient to the approximation of axial symmetry, because the gradient values inferred from measurements, as it was mentioned earlier, are comparable in both halves.

The rotation curve which we adopt in this work is shown in figure Fig.1. It is obtained by interpolating the measured rotation values and, additionally, by extending them beyond ≈ 17 kpc by a Keplerian tail. The purpose of this artificial extension is only to facilitate integration to infinity in finding the corresponding surface density, and it will not significantly affect the surface density at lower radii in the region of interest for the gradient estimation. This extension is not entirely arbitrary, however. A decrease in a Keplerian fashion in the rotation, in a neighborhood of the last measured point, was reported in another work investigating NGC 4244 (Olling 1996). This observation may be used to support the form of the artificial extension we adopted.

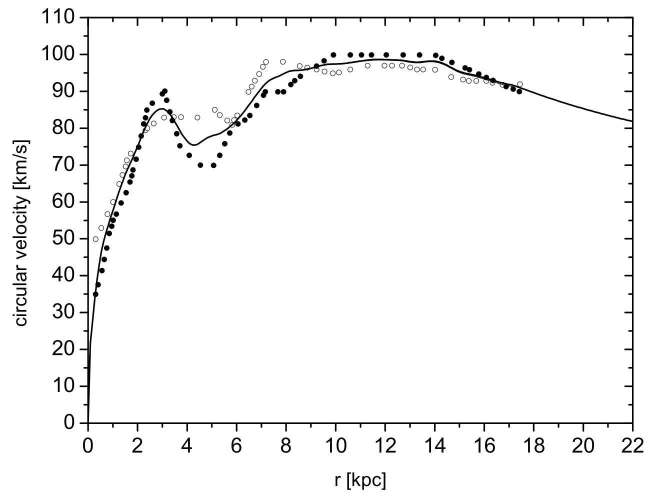

Figure 1. Circular velocity of NGC 4244 in the mid-plane as a function of the galactocentric distance (obtained by assuming a distance to NGC 4244 of 4.4 Mpc). The collection of points show the rotation values in the approaching *[black circles]* and receding *[open circles]* halves read from curves published in (Zschaechner et al. 2011). The *[solid line]* is our adopted rotation curve obtained by means of interpolating between the rotation values in both halves and extended beyond ≈ 17 kpc by adjoining a Keplerian tail.

2.2 Surface density in NGC 4244

In modeling the mass distribution and the vertical gradient, we limit ourselves to using the thin disk model, instead of applying a finite-width disk, because, as it was illustrated by Jałocha et al. (2014), when the gradient is calculated above the mid-plane, outside the main concentration of masses characterized by some vertical width-scale, then the vertical structure becomes insignificant for the gradient determination in this region and one can use a simpler model.

We determine a surface density in thin disk model based on a given circular velocity, by using a formula turning the velocity squared to the surface density as derived by

Bratek et al. (2014):

$$\sigma(\rho)=\frac{1}{2\pi^2 G}\int\limits_0^\infty\left[\frac{K\left(\frac{2\sqrt{x}}{1+x}\right)}{1+x}+\frac{E\left(\frac{2\sqrt{x}}{1+x}\right)}{1-x}\right]\frac{v^2(x^{-1}\rho)}{\rho}\,\mathrm{d}x. \quad (1)$$

This integral should be evaluated in the principal value sense about the singularity $x=1$ of the expression in the square brackets.[1]

As we have already pointed this out, the surface mass density is uncertain close to the radial position of the last point on the rotation curve and beyond it, owing to to the nature of a flattened mass distribution as such and lack of measurement data. In some cases we could overcome this indeterminacy by additionally using a surface density of gas distributed beyond the last point and find a global rotation by iterations in a way similar to that proposed by Jałocha et al. (2008). But there is no such complementary measurements available for NGC 4244, and we have to choose a different approach.

Since we are interested in the gradient modeling in the galactic interior $r<10\,\mathrm{kpc}$ separated well enough from the last measured point at $\approx 17\,\mathrm{kpc}$, we chose to extrapolate the rotation by smoothly joining to it a Keplerian tail, so as to extend the natural slope of the curve to larger radii, as shown in Fig.1. The internal part of the corresponding surface density is not significantly dependent on this extension to within reasonable distortions of that extension. To support this statement, we recall a calculation presented in (Bratek et al. 2008), which implies that in thin disk model the relative contribution to the local surface density from an almost Keplerian tail can be neglected at radii lower than roughly 2/3 of the radial extent of the measured part of the rotation curve, provided the mass function of the flattened mass component can be assumed to be almost saturated at the last measurement point. Then, in practice, it would make no difference if we set the rotation to be zero beyond the last measured point or not.

The surface density corresponding to the adopted rotation curve shown in Fig.1 is presented in Fig.2. It should be stressed that in disk model, unlike in spherical symmetry, the Keplerian tail in a region is not equivalent to lack of matter in that region, which is readily seen in Fig.2, where surface density is indeed non-vanishing in the region beyond $\approx 17\,\mathrm{kpc}$, where the rotation curve was set to be Keplerian.

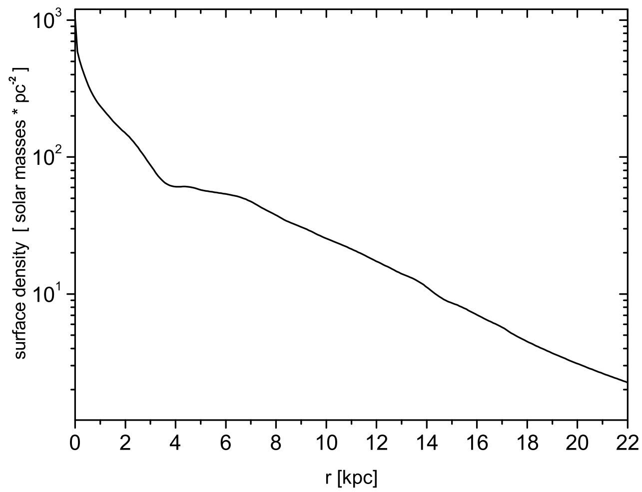

Figure 2. Surface mass density accounting in thin disk model for the adopted rotation curve of galaxy NGC 4244 shown in Fig.1. Unlike in spherical symmetry, surface density is nonzero in a region $r>17\,\mathrm{kpc}$ with Keplerian rotation curve.

3 VERTICAL GRADIENT IN AZIMUTHAL VELOCITY

In the quasi-circular orbits approximation (Jałocha et al. 2010), the azimuthal velocity at an altitude z above the mid-plane is found from

$$v_\varphi^2(r,z)=\int\limits_0^\infty\frac{2\,G\,\sigma(\chi)\,\chi\,\mathrm{d}\chi}{\sqrt{(r+\chi)^2+z^2}}\left(K\left[\kappa\right]-\frac{\chi^2-r^2+z^2}{(r-\chi)^2+z^2}\,E\left[\kappa\right]\right), \quad (2)$$

$$\kappa=\sqrt{\frac{4r\chi}{(r+\chi)^2+z^2}},$$

where the surface density expressed by Eq.1 have been substituted for $\sigma(\chi)$. This relation is used to describe the behavior of the azimuthal velocity component in a region above the mid-plane and exterior to the main concentration of masses of the flattened mass distribution, where the vertical structure of this distribution can be neglected, but still close enough to the mid-plane, so that the projection of orbits onto the mid-plane could be regarded as a perturbation of the concentric circular orbits based on which Eq.1 turning the circular velocity to the surface density (and its inverse) are derived in thin disk model (though, a reasoning in (Jałocha et al. 2010) admits a more general class of orbits).

We will estimate the vertical gradient in azimuthal velocity with the help of two methods. The first method gives a mean magnitude of the gradient by means of a linear approximation of the azimuthal velocity. The mean value, denoted by γ, is determined in a given rectangular region (r,z) by finding a best fitting profile of the form

$$v_\varphi(r,z)=v_\varphi(r,0)+\gamma\,|z|+\delta\,z, \quad (3)$$

approximating the values of azimuthal velocity stipulated at several points arranged in an array in that region. The linear formula was used by Levine et al. (2008) to estimate the vertical fall-off rate in the rotation speed of the Milky-Way galaxy. Here, the use of the linear expansion is justified by the linearity of the decrease in azimuthal velocity predicted by the thin disk model for NGC 4244, as illustrated in Fig.3. Owing to this behavior (and the reflection symmetry with respect to the mid-plane $z=0$, which implies that the rolling parameter δ must be set equal to zero in disk model), it

[1] We use complete elliptic integrals K and E in forms as defined by Gradshtein et al. (2007):

$$K(k)=\int\limits_0^{\pi/2}\frac{\mathrm{d}\phi}{\sqrt{1-k^2\sin^2\phi}},\qquad E(k)=\int\limits_0^{\pi/2}\mathrm{d}\phi\sqrt{1-k^2\sin^2\phi}.$$

Note, that they differ in the adopted convention for the argument notation from those given by Abramowitz & Stegun (1972).

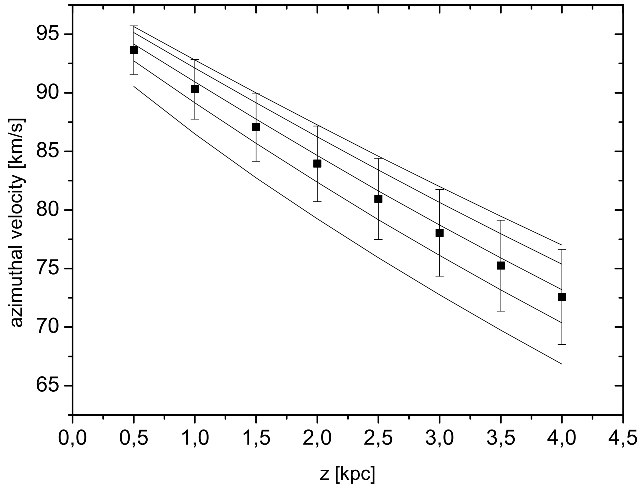

Figure 3. Behavior of azimuthal velocity off the mid-plane in galaxy NGC 4244, predicted in quasi-circular orbits approximation in thin disk model (that is, calculated with the help of integral Eq.2). The thin lines are curves of constant radial variable: $r = 8, 9, 10, 11, 12\,\mathrm{kpc}$, counting from the bottom line to the top line. Each point represents the mean azimuthal velocity component, with the respective bar representing the standard deviation. The velocity decreases almost linearly with the altitude $|z|$ above the mid-plane.

	δr		δz	$\frac{\partial v_\phi}{\partial z}\ [\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}]$
$r\in(1,16)$	0.2	$z\in(0.5,1)$	0, 02	-8.38 ± 0.33
$r\in(2,10)$	0.2	$z\in(0.5,3)$	0.2	-8.24 ± 0.21
$r\in(10,16)$	0.2	$z\in(0.5,1)$	0.02	-5.23 ± 0.05
$r\in(10,11)$	0.01	$z\in(0.5,3)$	0.2	-6.00 ± 0.00

Table 1. Rectangular arrays of rotation data (z, r , δz and δr, are expressed in kpc) and the corresponding mean vertical gradient of rotation (coefficient γ in Eq.3).

is rational to consider only a single number describing the vertical velocity fall-off for NGC 4244, namely, the mean gradient magnitude γ.

Table 1 shows the results of applying the first method to the theoretical rotation values predicted by evaluating the integral Eq.2 inside several rectangular regions defined in that table. To some degree, the gradient results depend on the horizontal and vertical extent of a given region where the gradient is estimated. To trace this dependence we use a second method.

The second method of determining the vertical gradient in azimuthal velocity uses a direct expression for the gradient, obtained by differentiating the expression for velocity Eq.2 with respect to z and using some identities between elliptic integrals K, E and their derivatives. The result is

$$\partial_z v_\varphi\,(r,z) = \frac{Gz}{v_\varphi(r,z)}\int\limits_0^\infty \frac{\chi\sigma(\chi)\mathrm{d}\chi}{\left(z^2+(r+\chi)^2\right)^{\frac{3}{2}}}\times \tag{4}$$

$$\left(\frac{r^2-z^2-\chi^2}{z^2+(r-\chi)^2}K(\kappa)-\frac{7r^4+6r^2\left(z^2-\chi^2\right)-\left(z^2+\chi^2\right)^2}{\left(z^2+(r-\chi)^2\right)^2}E(\kappa)\right).$$

Fig.4 illustrates the gradient dependence in function of the

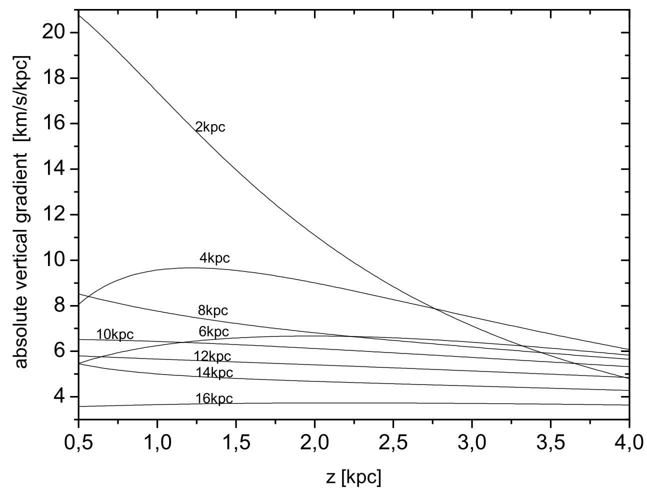

Figure 4. Absolute values of the vertical gradient in azimuthal velocity in thin disk model shown in function of the altitude z above the mid-plane for various values of the radial variable from within the interval $r \in (2, 16)$ kpc in steps of 2 kpc. These lines represent the values obtained using the integral expression Eq.4.

altitude above the mid-plane for various radii in a region from 2 kpc to 16 kpc. The absolute gradient value depends weakly on the z variable, except close to the galactic center. This reflects the linear decrease of azimuthal velocity evident in Fig.3. The maximum gradient magnitude exceeds even $20\,\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ but only close to the center and at low heights above the mid-plane. In a prevailing part of the galactic region the gradient magnitude does not exceed $10\,\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$, decreasing moderately with the radial distance and is lower than $4\,\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ for the outermost radii (compare, the $r = 16\,\mathrm{kpc}$ line in Fig.4).

Fig.5 shows for several altitudes the variation in the

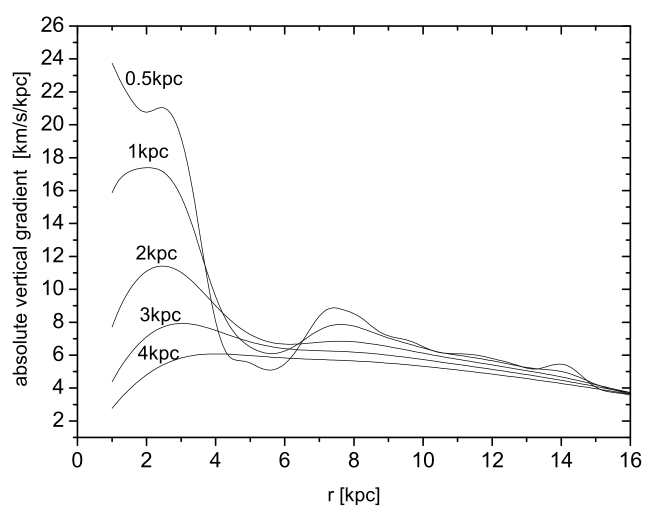

Figure 5. Absolute values of the vertical gradient in azimuthal velocity shown as functions of the radial variable for various altitudes above the mid-plane.

gradient in function of the radial distance. In the central

part the absolute gradient value is not only high (exceeding 20 $\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ for low altitudes) but it also strongly changes with the radial variable. But starting from a radius of 4 kpc the gradient magnitudes are much lower (below 10 $\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$) and they exhibit a significantly weaker dependence on the radial variable, being more pronounced only for lower altitudes above the mid-plane.

We can summarize the above results by saying that the vertical gradient predicted for NGC 4244 is not too high – its mean value in a rectangular region $r \in (1, 16)$ kpc and $|z| \in (0.5, 1)$ kpc is $-8.38 \pm 0.33 \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$, and it would not differ much from a value of $-8.24 \pm 0.21 \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ obtained by shrinking the radial interval to $r \in (2, 10)$ kpc and increasing the height above the mid-plane to 3 kpc. We predict also a decrease in the gradient magnitude for higher radial distances: in a rectangle $r \in (2, 16)$ kpc and $|z| \in (0.5, 1)$ kpc we obtain a mean gradient value of $-5.23 \pm 0.05 \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$. These predictions are in agreement to within error limits with the gradient properties reported in (Zschaechner et al. 2011), namely, with the gradient value of $-9^{+3}_{-2} \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ in the approaching half and $-9^{+2}_{-2} \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ in the receding half, and with the gradient magnitude found to be decreasing to $5^{+2}_{-2} \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ and $4^{+2}_{-2} \frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$ near a radius of 10 kpc in the approaching and receding halves, respectively.

In the first method we defined the error of our model gradient value as the uncertainty in the parameter γ inferred from a linear regression model Eq.3 fitting to the velocity values predicted for quasi-circular orbits. The error does not include the error transferred from the uncertainties in the rotation curve determination. We can get a good impression of the scale of these uncertainties from Fig.6 which illustrates the influence of variations in the rotation curve on the predicted gradient values. But the relative errors in the gradient determination in (Zschaechner et al. 2011) are even higher, therefore a more precise analysis of the model uncertainties seems of minor importance for our conclusions concerning the gradient properties.

4 CONCLUDING REMARKS

There are various models aimed at explaining the presence and behavior of the vertical gradient of azimuthal velocity (e.g., Collins et al. (2002); Fraternali & Binney (2006)), but our model is characterized by high simplicity and minimum of assumptions. We use a disk model with a surface density accounting for the full rotation curve measured for a galaxy, assuming that the motion of matter is entirely due to the gravitational potential of the disk. This means that most of matter is assumed to form a flattened mass distribution, without a dominating heavy halo. Despite this simplicity our model accounts for the vertical gradient in azimuthal velocity, predicting correct gradient magnitudes and its approximate independence of the altitude above the mid-plane.

NGC 4244 is the next among the galaxies examined by us so far (Jałocha et al. (2010, 2011)) for which our approach to modeling the gradient turned out to perform satisfactorily well, predicting vertical gradient magnitudes and their behavior consistently with expectations based on the measurements. The most controversial in this approach may be our not including the other mass components except the disk-like only. But obtaining satisfactory results in modeling a galaxy in this approach, simply may indicate that the galaxy has a dominating flattened mass component, at least in the region where the gradient is measured and modeled. This we give as the main conclusion of our work.

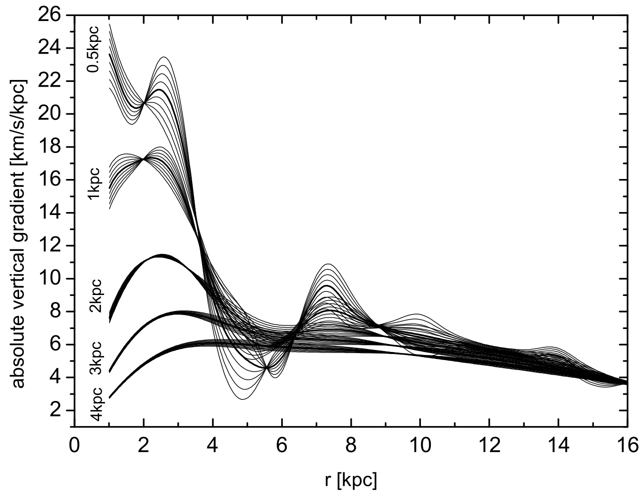

Figure 6. This figure shows results analogous to those in Fig.5 for various rotation curves. Thick lines correspond to a rotation curve obtained by smoothing spline interpolating between the approaching- and receding- half rotation curves (it almost overlaps the adopted rotation curve in Fig.1, which shows that the averaging method is not influential to the overall shape of that curve). Thin lines correspond to deformed rotation curves obtained by taking linear combinations $\alpha v_a + (1-\alpha) v_r$ of the rotation curve in the approaching half (denoted here by v_a) and that in the receding half (denoted by v_r).

An interesting property we want to bring to the attention are high gradient magnitudes in the galactic center vicinity of NGC 4244 (mainly close to the mid-plane), exceeding the mean gradient magnitude determined for this galaxy. A similar phenomenon occurs for another galaxy. Zschaechner et al. (2011) mention that the measured gradient magnitude in the central part of NGC 891 is very high (reaching 43 $\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$) although it decreases strongly for outer radii (to 14 $\frac{\mathrm{km}}{\mathrm{s\cdot kpc}}$). This may suggest that high gradient magnitudes in the central parts of spiral galaxies (much higher than the mean magnitude for the whole disk) might be a qualitative property of the vertical gradient which the disk model accounts for well.

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